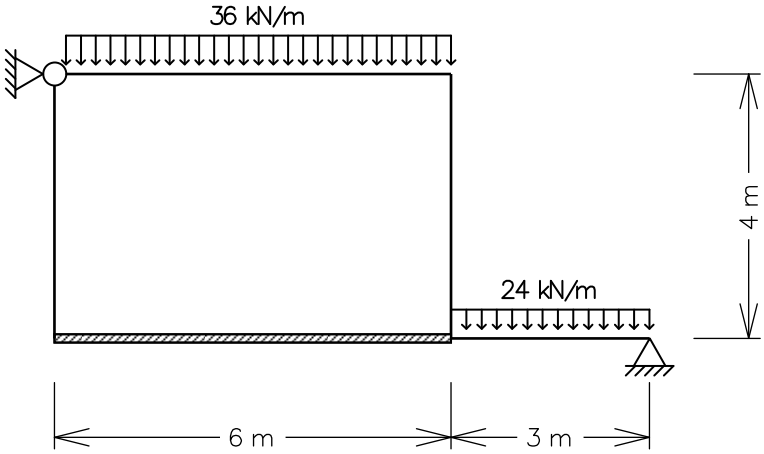
ENG 1204 – ANÁLISE DE ESTRUTURAS II – 1º Semestre – 2018

Segunda Prova –Datas: 30/05/2018 – 04/06/2018 – Sem Consulta

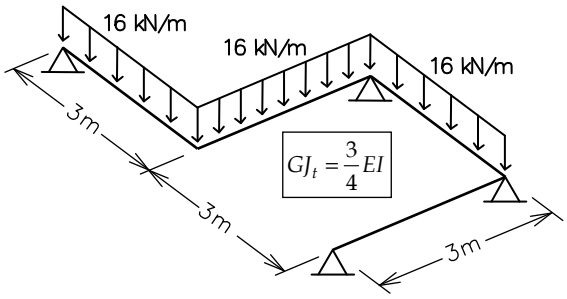
1ª Questão (5,5 pontos)

Empregando-se o Método dos Deslocamentos, obter o diagrama de momentos fletores para o quadro abaixo (barras inextensíveis). Todas as barras têm a mesma inércia à flexão EI = 3.6x10^4 kNm^2, com exceção da barra horizontal inferior da esquerda, que é infinitamente rígida à flexão.



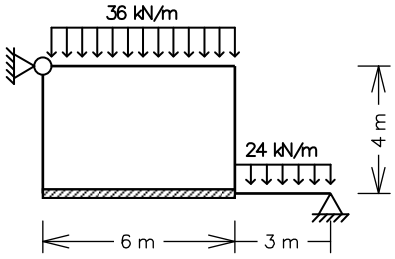
2ª Questão (3,5 pontos)

Empregando-se o Método das Forças, obter os diagramas de momentos fletores e momentos torçores para a grelha abaixo. Todas as barras têm a relação indicada entre a rigidez à torção GJ_t e a rigidez à flexão EI.

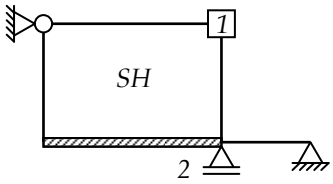


Solução de um sistema de 2 equações a 2 incógnitas: $\begin{Bmatrix} e \\ f \end{Bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} D_1 = \dfrac{bf - de}{ad - bc} \\ D_2 = \dfrac{ce - af}{ad - bc} \end{cases}$

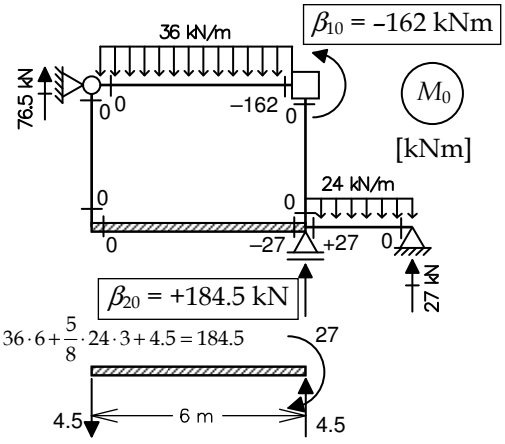
1ª Questão



Sistema Hipergeométrico

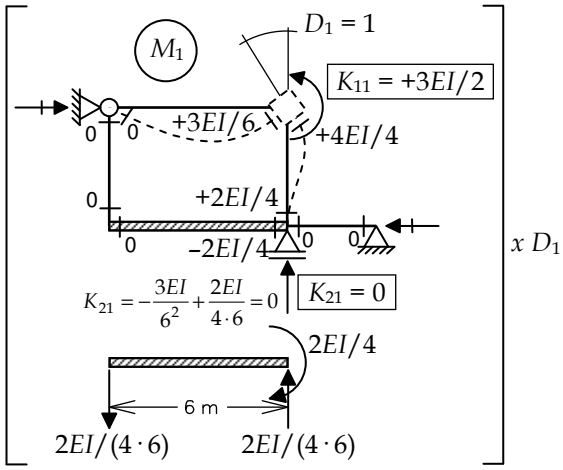


Caso (0) – Solicitação externa isolada no SH



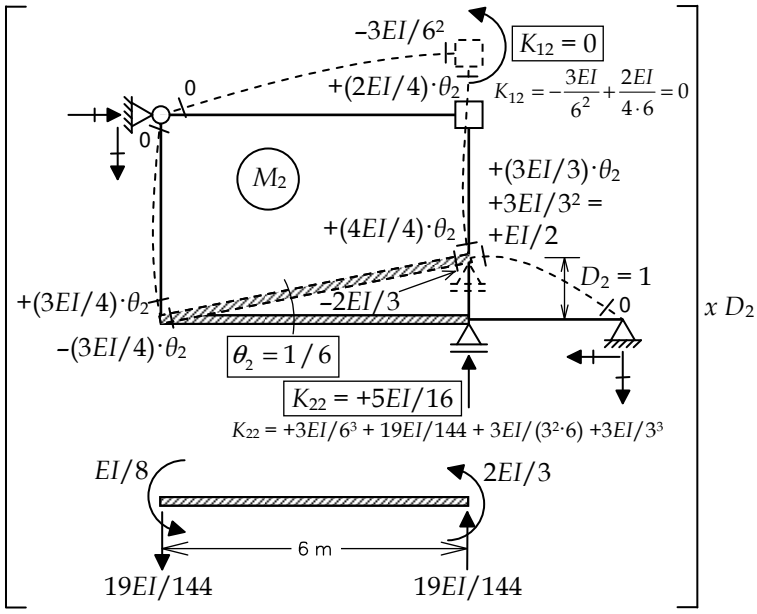
$$\beta_{20} = \frac{5}{8}\cdot 36\cdot 6 + \frac{5}{8}\cdot 24\cdot 3 + 4.5 = 184.5$$

Caso (1) – Deslocabilidade D_1 isolada no SH



$$K_{21} = -\frac{3EI}{6^2} + \frac{2EI}{4\cdot 6} = 0$$

Caso (2) – Deslocabilidade D_2 isolada no SH



$$K_{12} = -\frac{3EI}{6^2} + \frac{2EI}{4\cdot 6} = 0$$

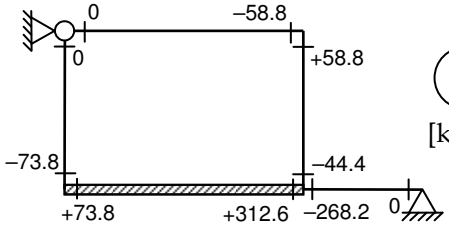
$$K_{22} = +3EI/6^3 + 19EI/144 + 3EI/(3^2\cdot 6) + 3EI/3^3$$

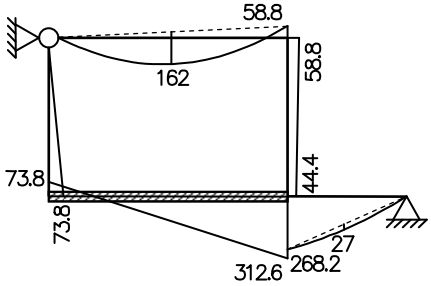
Equações de equilíbrio:

$$\begin{cases} \beta_{10} + K_{11}D_1 + K_{12}D_2 = 0 \\ \beta_{20} + K_{21}D_1 + K_{22}D_2 = 0 \end{cases} \Rightarrow \begin{Bmatrix} -162 \\ +184.5 \end{Bmatrix} + EI\cdot\begin{bmatrix} +3/2 & 0 \\ 0 & +5/16 \end{bmatrix}\cdot\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} D_1 = +\dfrac{108}{EI} \\ D_2 = -\dfrac{590.4}{EI} \end{cases}$$

Momentos Fletores Finais:

$$M = M_0 + M_1\cdot D_1 + M_2\cdot D_2$$

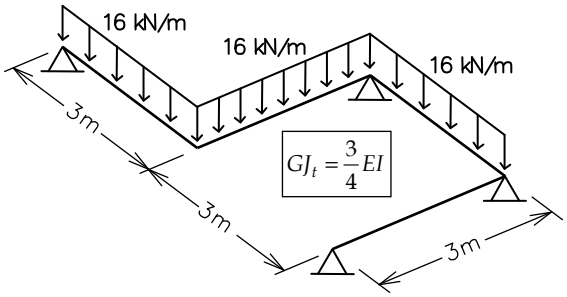




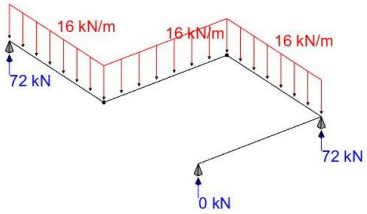
M

[kNm]

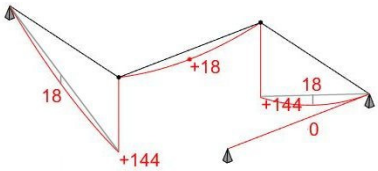
2ª Questão



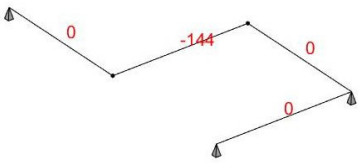
Caso (0) – Solicitação externa isolada no Sistema Principal



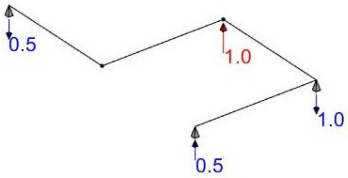
Momentos fletores (kNm)



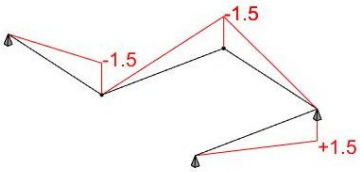
Momentos de torção (kNm)



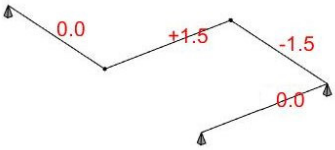
Caso (1) com $X_1 = 1$ – Hiperestático X_1 com valor unitário



Momentos fletores (kNm/m)



Momentos de torção (kNm/m)



Equação de compatibilidade

$$\delta_{10} + \delta_{11} X_1 = 0$$

$$\delta_{10} = \left[-\frac{1}{3}\cdot 1.5\cdot 144\cdot 3 - \frac{1}{3}\cdot 1.5\cdot 18\cdot 3 - \frac{1}{3}\cdot 1.5\cdot 18\cdot 3 - \frac{1}{3}\cdot 1.5\cdot 144\cdot 3 - \frac{1}{3}\cdot 1.5\cdot 18\cdot 3 \right]\cdot \frac{1}{EI} +$$

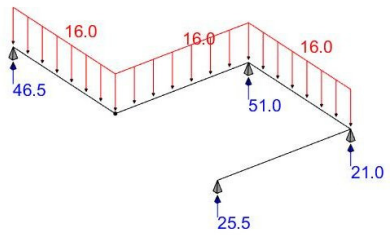
$$\left[-1.5\cdot 144\cdot 3\right]\cdot \frac{1}{GJ_t} = -\frac{513}{EI} - \frac{648}{GJ_t} = -\frac{513}{EI} - \frac{4}{3}\cdot\frac{648}{EI} = -\frac{1377}{EI}$$

$$\delta_{11} = \left[4\cdot\left(+\frac{1}{3}\cdot 1.5\cdot 1.5\cdot 3 \right)\right]\cdot\frac{1}{EI} + \left[(+1.5)\cdot(+1.5)\cdot 3 + (-1.5)\cdot(-1.5)\cdot 3\right]\cdot\frac{1}{GJ_t} =$$

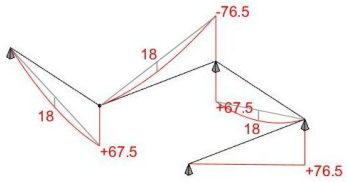
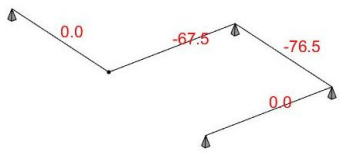
$$+\frac{9}{EI} + \frac{13.5}{GJ_t} = +\frac{9}{EI} + \frac{4}{3}\cdot\frac{13.5}{EI} = +\frac{27}{EI}$$

$$\Rightarrow -\frac{1377}{EI} + \frac{27}{EI}\cdot X_1 = 0 \qquad \boxed{\therefore X_1 = +51\text{ kN}}$$

Superposição de casos básicos: Caso (0) + [Caso (1) com $X_1 = 1$] x X_1



Momentos fletores (kNm)



Momentos de torção (kNm)